Table 3. Comparison of  $4\cdot2^\circ K$  elastic constants and adiabatic bulk modulus in units of  $10^{11}$  dyne cm $^{-2}$ 

| Investigator        | C11   | $C_{12}$ | C44   | $B_s$ |
|---------------------|-------|----------|-------|-------|
| Authors             | 5.834 | 1.192    | 1.337 | 2.739 |
| Lewis et al.(8)     | 5.733 | 1.123    | 1.331 | 2.660 |
| Overton and Swim(3) | 5.750 | 0.986    | 1.327 | 2.574 |

power series with the leading term being proportional to  $T^4$ , which gives a  $T^3$  leading term to the linear expansion coefficient. Coupling this result with the Debye temperature,  $\theta_D$ , the molar volume, V, and the bulk modulus,  $B_s$ , he calculates the Gruneisen parameter  $\gamma_0$  corresponding to the low temperature  $T^3$  region. White used the bulk modulus determined by OVERTON and SWIM<sup>(3)</sup> with the result that  $\gamma_0 = 0.90 \pm 0.03$ . The present measurement of the bulk modulus is 6.6% higher than Overton and Swim's value and increases  $\gamma_0$  to  $0.96 \pm 0.03$ .

In terms of the microscopic parameters (10) of the crystal, the Gruneisen parameter is given by

$$\gamma = \frac{\sum_{i} c_{i} \gamma_{i}}{\sum_{i} c_{i}},$$
 (5)

where  $c_i$  is the Einstein heat capacity of the  $i^{th}$  normal mode. In the  $T^3$  region of temperature, one may assume the crystal to behave like an elastic continuum with the weighting factors in the above expression being replaced by  $C_i^{-3/2}$ , the elastic constant for the  $i^{th}$  branch in the direction  $\theta$ ,  $\phi$ , and the above equation goes over to the

following integral form

$$\bar{\gamma}_L = \frac{\sum_{i=1}^{3} \int_{\Omega} \gamma_i C_i^{-3/2} d\Omega}{\sum_{i=1}^{3} \int_{\Omega} \gamma_i C_i^{-3/2} d\Omega}$$
(6)

where  $d\Omega$  is the element of solid angle. The  $\gamma_i$  for low frequency modes can be calculated from the pressure derivatives of the elastic constants as described by various authors. (10) The above integral has been calculated for temperatures of 295°K and 195°K using the low temperature elastic constants reported in Table 3 along with the pressure data of BARTELS. (4) These results, along with a volume extrapolation to 0°K as predicted by the Quasi Harmonic Approximation, are listed in Table 4. Bartels obtained an extrapolated value of 1.05, using Overton and Swim's low temperature elastic data, as compared to the authors' value of 1.06. There are two main reasons why  $\bar{\gamma}_L$  was insensitive to the changes in the low temperature elastic constants. First, the weighting factors occur in both the numerator and denominator in the expression for  $\bar{\gamma}_L$  and thus a slight change

Table 4.  $\bar{\gamma}_L$  for NaCl calculated from the mode gammas compared with White's Thermodynamic value of  $\gamma_0$ 

|   | 295°K | 195°K  | 0°K    | White     |
|---|-------|--------|--------|-----------|
| $\bar{\gamma}_L$                            | 1.17  | 1.12   | 1.06   | 0.96±0.03 |
| $rac{ar{\gamma}_L}{V_{295} - V} = V_{295}$ | 0     | 0.0112 | 0.0232 |           |

The values of  $\bar{\gamma}_L$  listed under 295°K and 195°K were calculated from the mode gammas at those temperatures and were 'volume extrapolated' to give the value listed under 0°K. This latter value can be considered to be calculated from the 0°K mode gammas and is the one that should be compared with White's value. Also given are the relative volume changes that were used. V is the volume at the temperature under consideration and  $V_{295}$  is the volume at 295°K.

in them tends to cancel. Second, the transverse mode corresponding to  $C_{44}$  carries the largest weighting since it is the lowest velocity mode, and the measured value of  $C_{44}$  reported by Overton and Swim differs with the authors' value by less than one per cent.

The Debye temperature,  $\theta_D$ , can be calculated from the value of the integral which appears in the denominator of the expression for  $\bar{\gamma}_L$ . The data of Overton and Swim gives a value of  $322 \cdot 3^{\circ} \text{K}$ , while the present data yields a value of  $322 \cdot 0^{\circ} \text{K}$ . This slight effect upon  $\theta_D$  is due to the fact that the dominant mode in determining  $\theta_D$  is  $C_{44}$ .

The main effect of the present measurements has been to increase the low temperature bulk modulus and also the measured low temperature Gruneisen parameter by about 7%, while leaving the calculated Gruneisen parameter,  $\bar{\gamma}_L$ , essentially unchanged.

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Note added in proof—P. P. M. MEINCKE and G. M. GRAHAM [Can. J. Phys. 43, 1853 (1965)] recently reported thermal expansion data on NaCl. Their technique involves the use of a Fabrey–Perot etalon dilatometer and for NaCl in the region below 12°K they fitted their experimental points with  $\alpha = 6.1 \pm 0.1$   $T^3$ /°K; which results in a  $\gamma_0 = 1.06$ . Using the authors' bulk modulus data raises their value of  $\gamma_0$  to 1.13.